Center for Causal Discovery

Day 2: Search

June 9, 2015

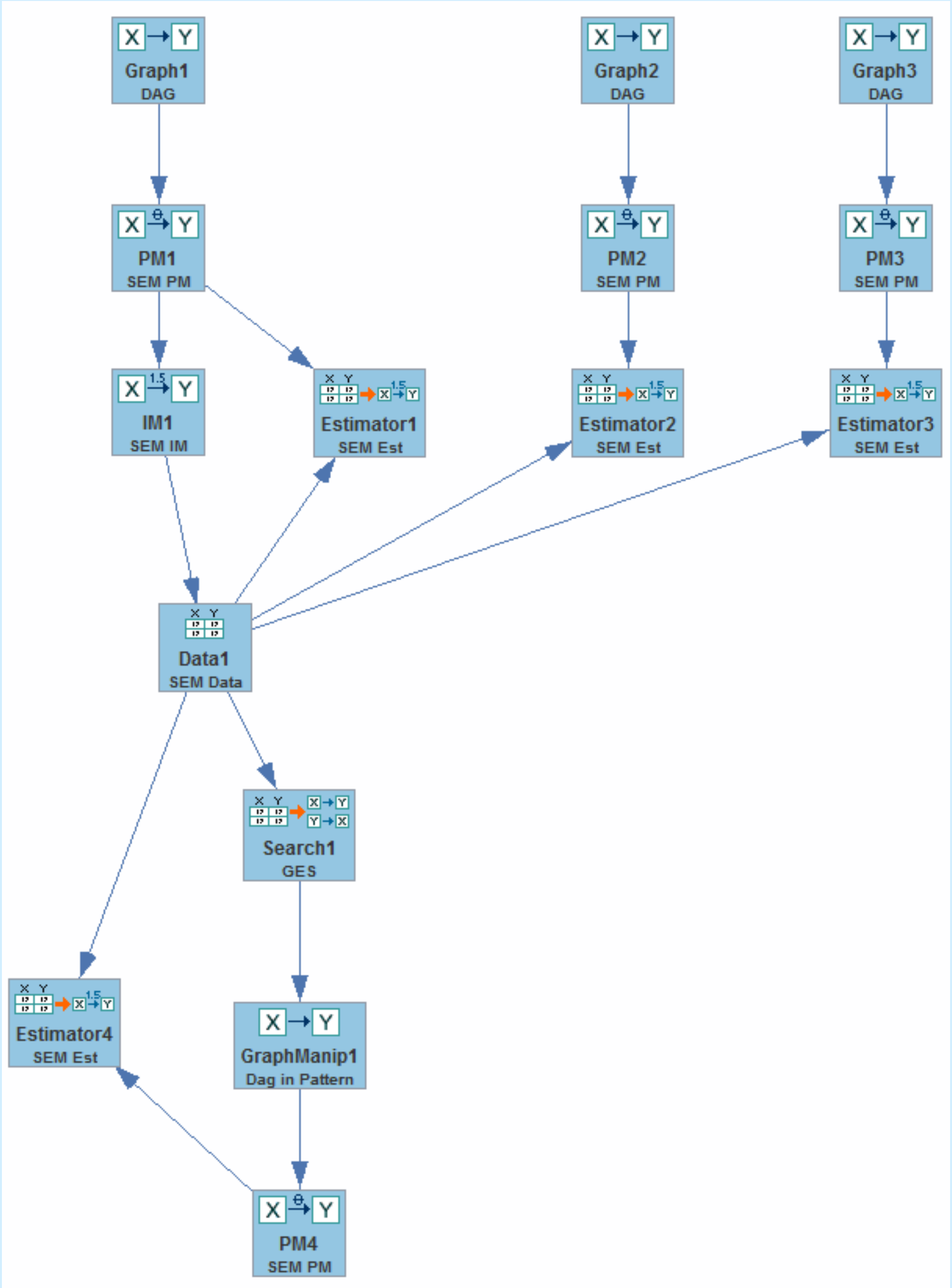
Carnegie Mellon University

Outline

Day 2: Search

1. Bridge Principles: Causation ←→ Probability
2. D-separation
3. Model Equivalence
4. Search Basics (PC, GES)
5. Latent Variable Model Search (FCI)
6. Examples

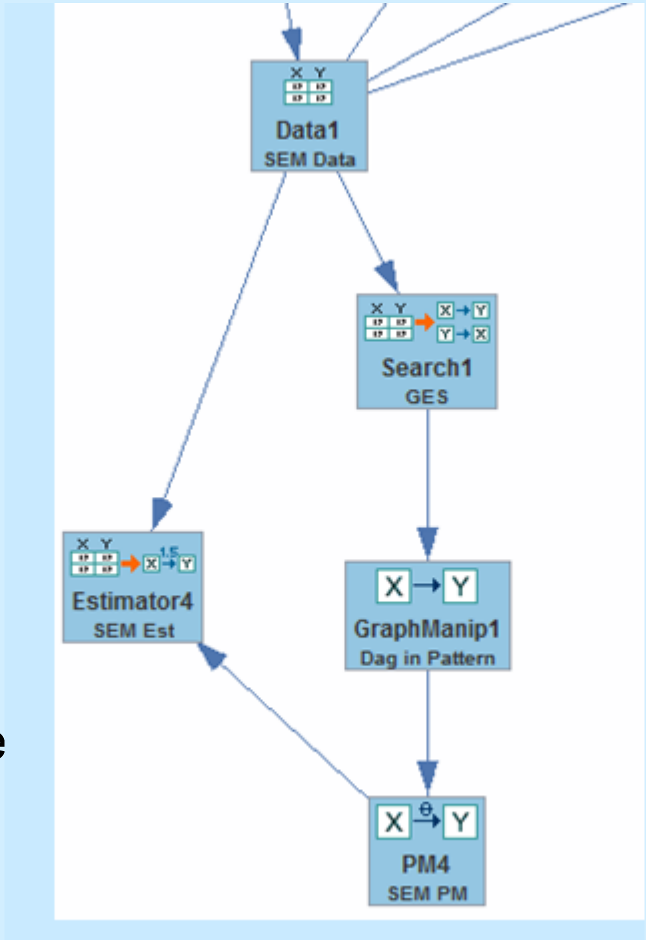
Tetrad Demo and Hands On



Tetrad Demo and Hands-on

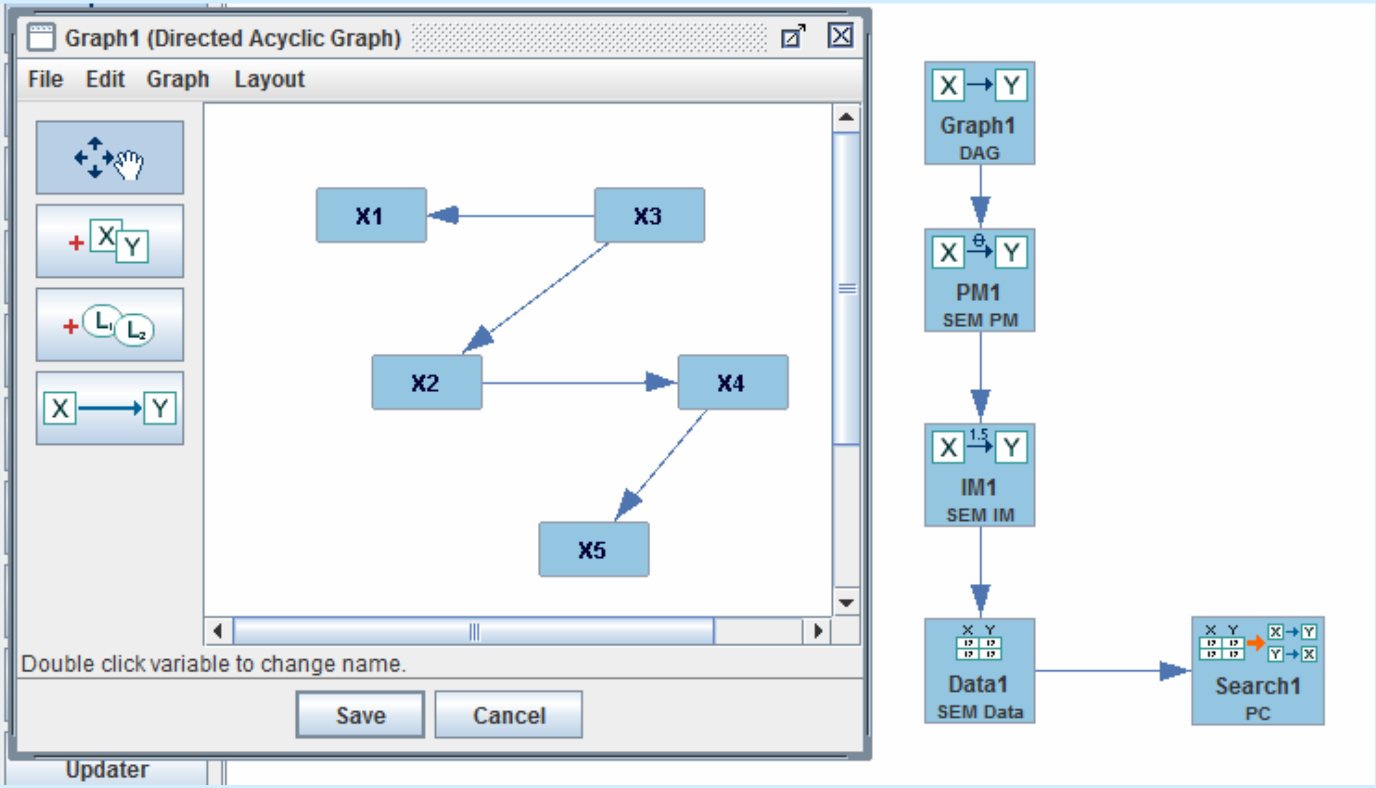
1) Go to “estimation2”
2) Add Search node (from Data1)

   - Choose and execute one of the “Pattern searches”
3) Add a “Graph Manipulation” node to search result: “choose Dag in Pattern”
4) Add a PM to GraphManip
5) Estimate the PM on the data
6) Compare model-fit to model fit for true mode



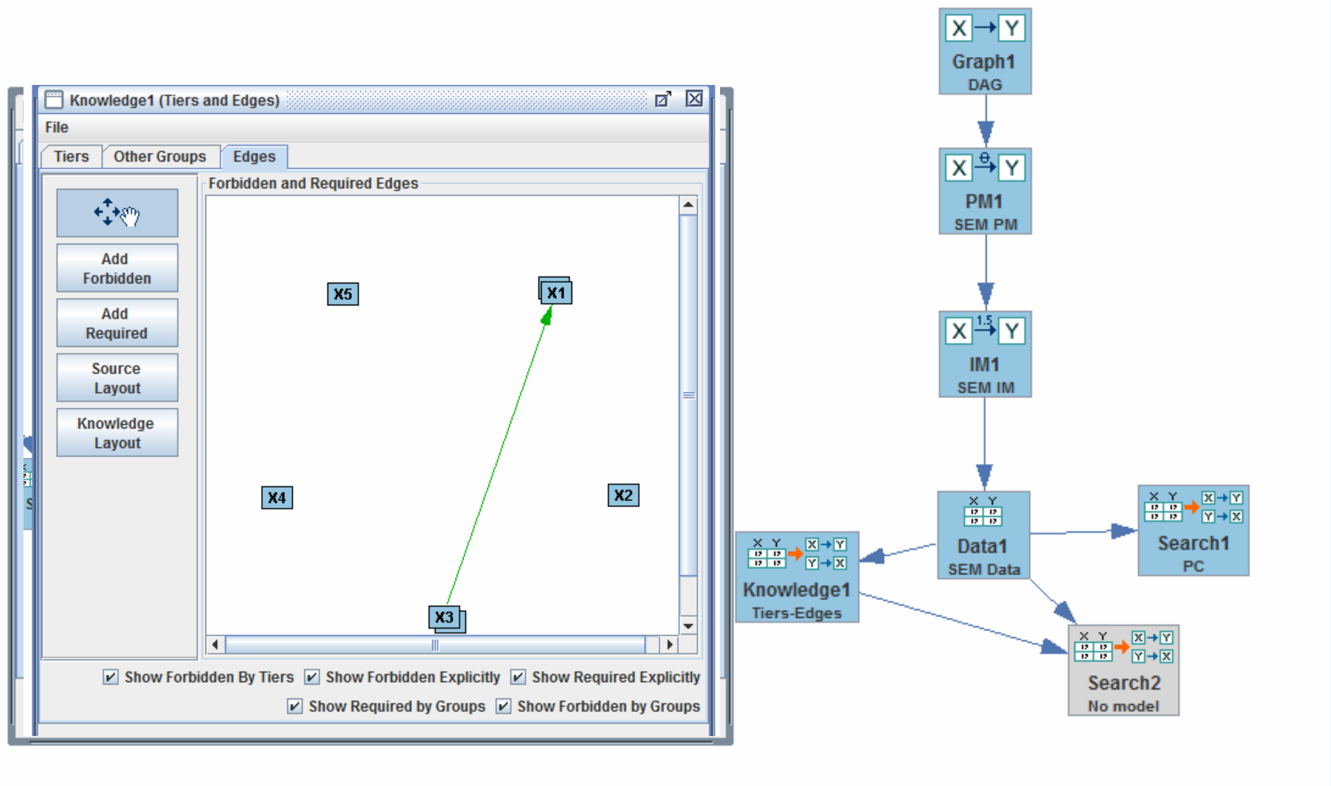
Backround Knowledge
Tetrad Demo and Hands-on

1) Create new session
2) Select “Search from Simulated Data” from Template menu
3) Build graph below, PM, IM, and generate sample data N=1,000.
4) Execute PC search, α = .05



Backround Knowledge
Tetrad Demo and Hands-on

1) Add “Knowledge” node – as below
2) Create “Required edge X3 → X1 as shown below.
3) Execute PC search again, α = .05
4) Compare results (Search2) to previous search (Search1)



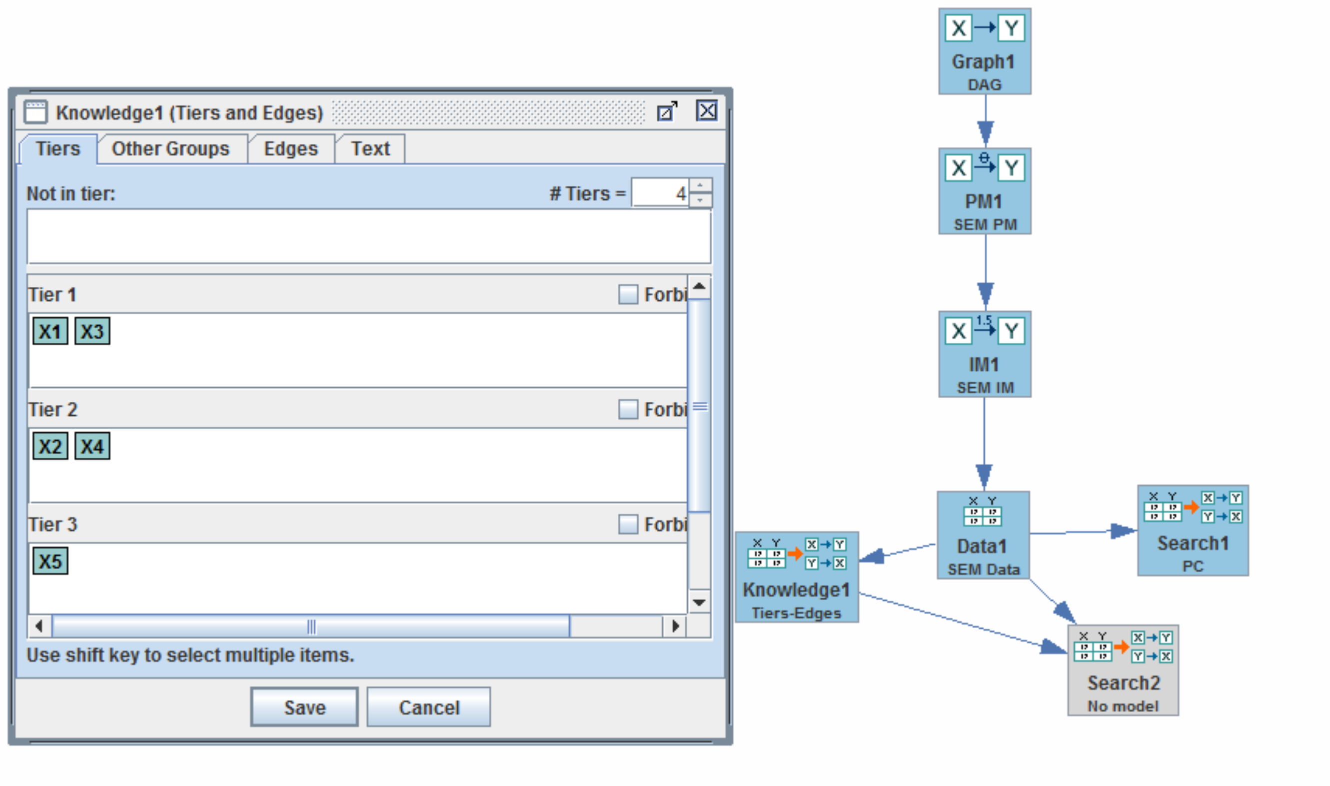
Backround Knowledge
Tetrad Demo and Hands-on

1) Add new “Knowledge” node

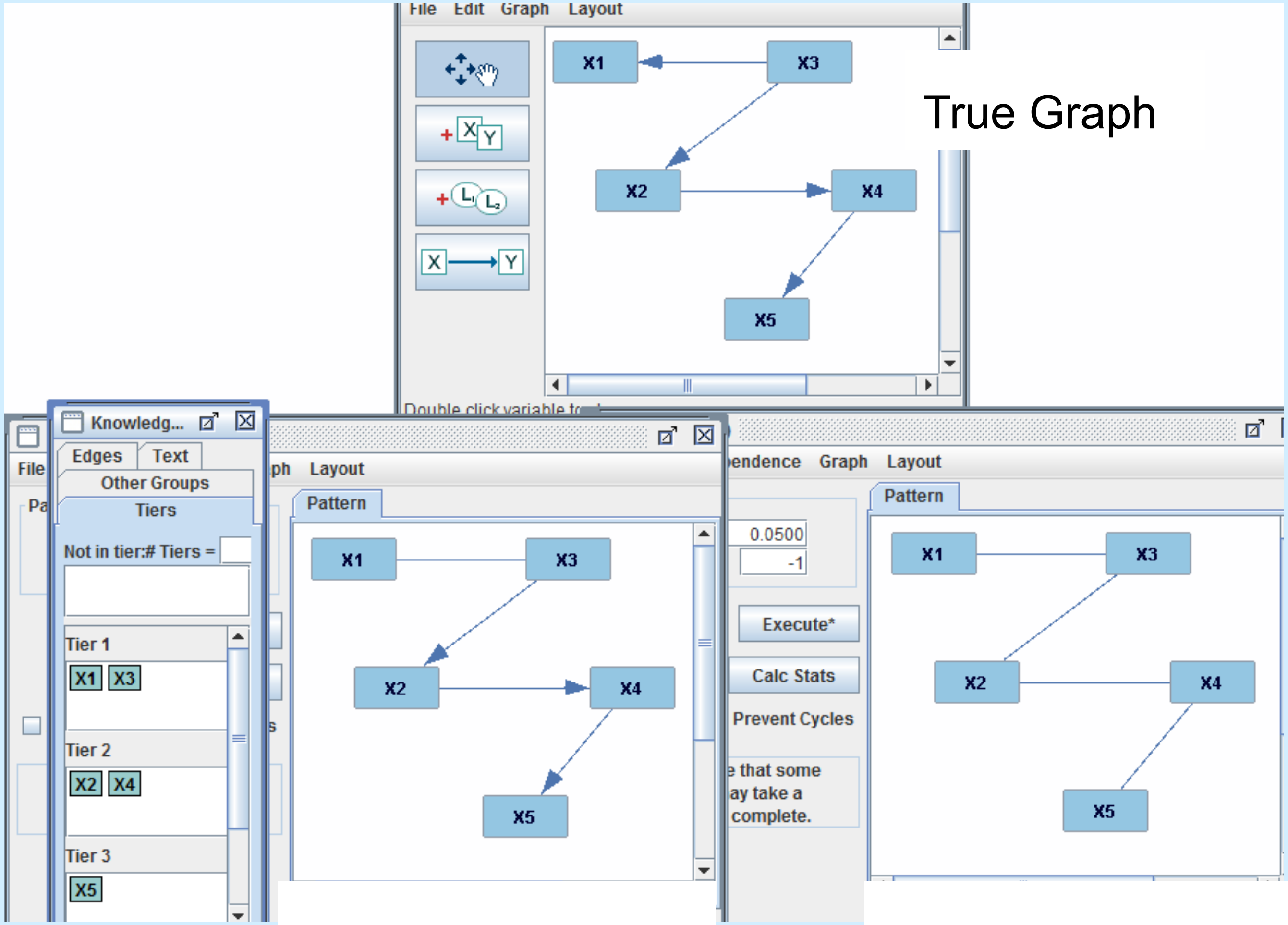
2) Create “Tiers” as shown below.

3) Execute PC search again, α = .05

4) Compare results (Search2) to previous search (Search1)



Backround Knowledge
Direct and Indirect Consequences



True Graph

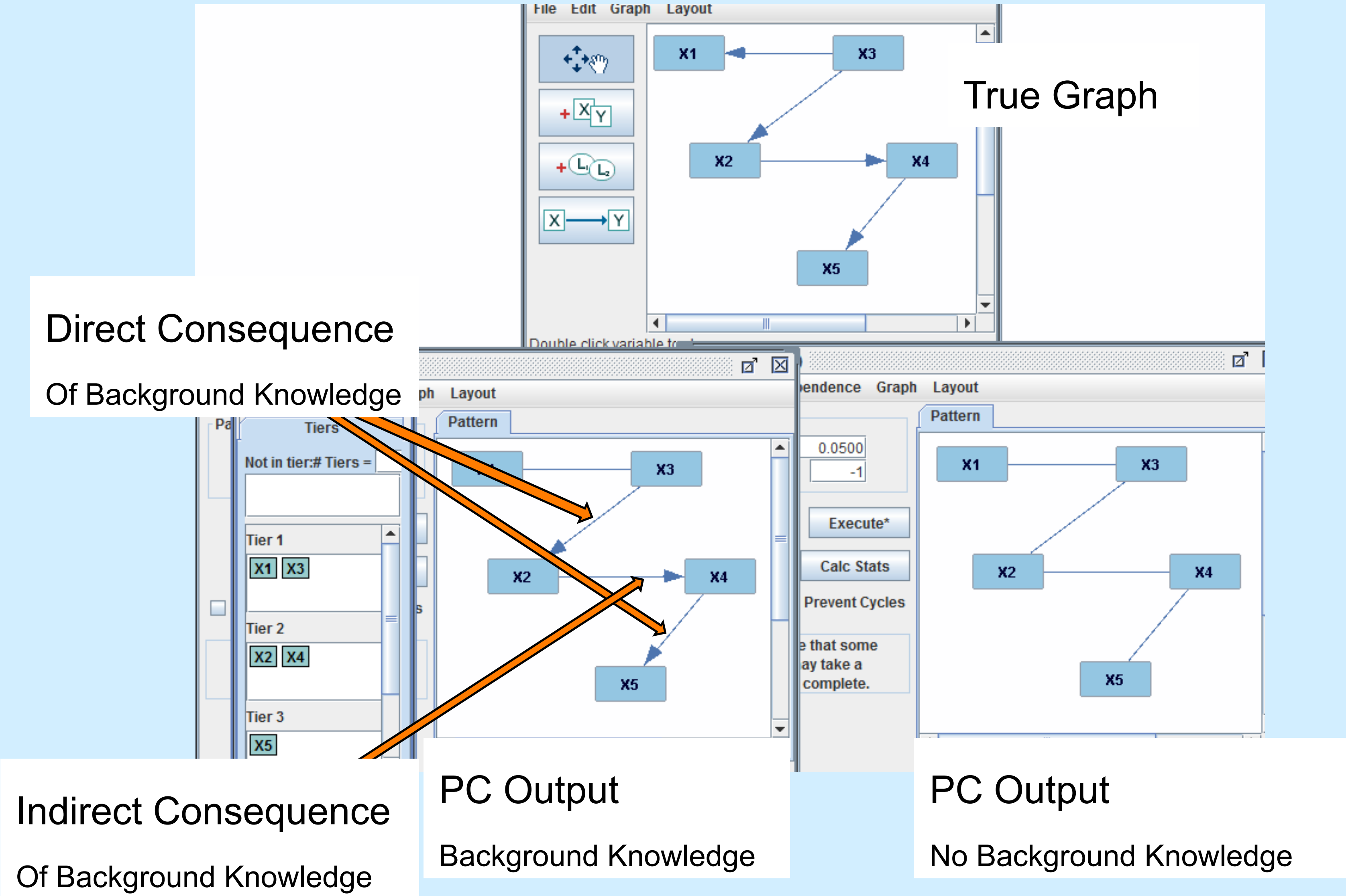
PC Output

Background Knowledge

PC Output

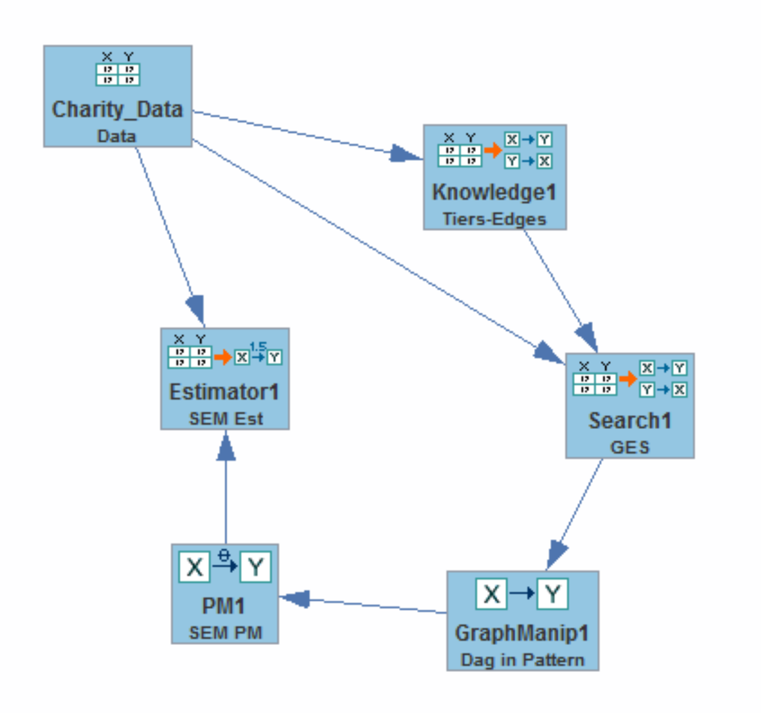
No Background Knowledge

Backround Knowledge
Direct and Indirect Consequences



Charitable Giving (Search)

1) Load in charity data
2) Add search node
3) Enter Background Knowledge:
   - Tangibility is exogenous
   - Amount Donated is endogenous only
   - Tangibility → Imaginability is required
4) Choose and execute one of the "Pattern searches"
5) Add a "Graph Manipulation" node to search result: "choose Dag in Pattern"
6) Add a PM to GraphManip
7) Estimate the PM on the data
8) Compare model-fit to hypothetical model



Constraint-based Search for Patterns

1) Adjacency phase

2) Orientation phase

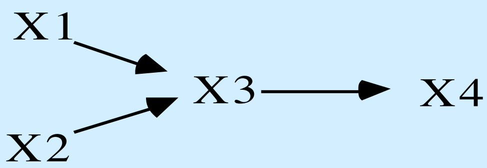
Constraint-based Search for Patterns: Adjacency phase

X and Y are not adjacent if they are independent conditional on any subset that doesn't X and Y

1) Adjacency

- Begin with a fully connected undirected graph
- Remove adjacency X-Y if X _||_ Y | any set S

Causal Graph



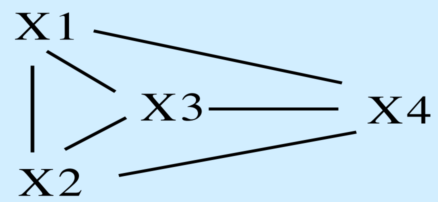
Independencies

$X1 \perp\!\!\!\perp X2$

$X1 \perp\!\!\!\perp X4 \mid \{X3\}$

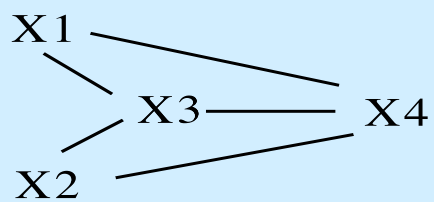
$X2 \perp\!\!\!\perp X4 \mid \{X3\}$

Begin with:



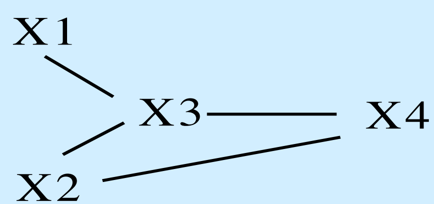
From

$X1 \perp\!\!\!\perp X2$



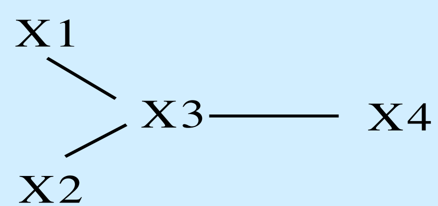
From

$X1 \perp\!\!\!\perp X4 \mid \{X3\}$



From

$X2 \perp\!\!\!\perp X4 \mid \{X3\}$



Constraint-based Search for Patterns: Orientation phase

2) Orientation

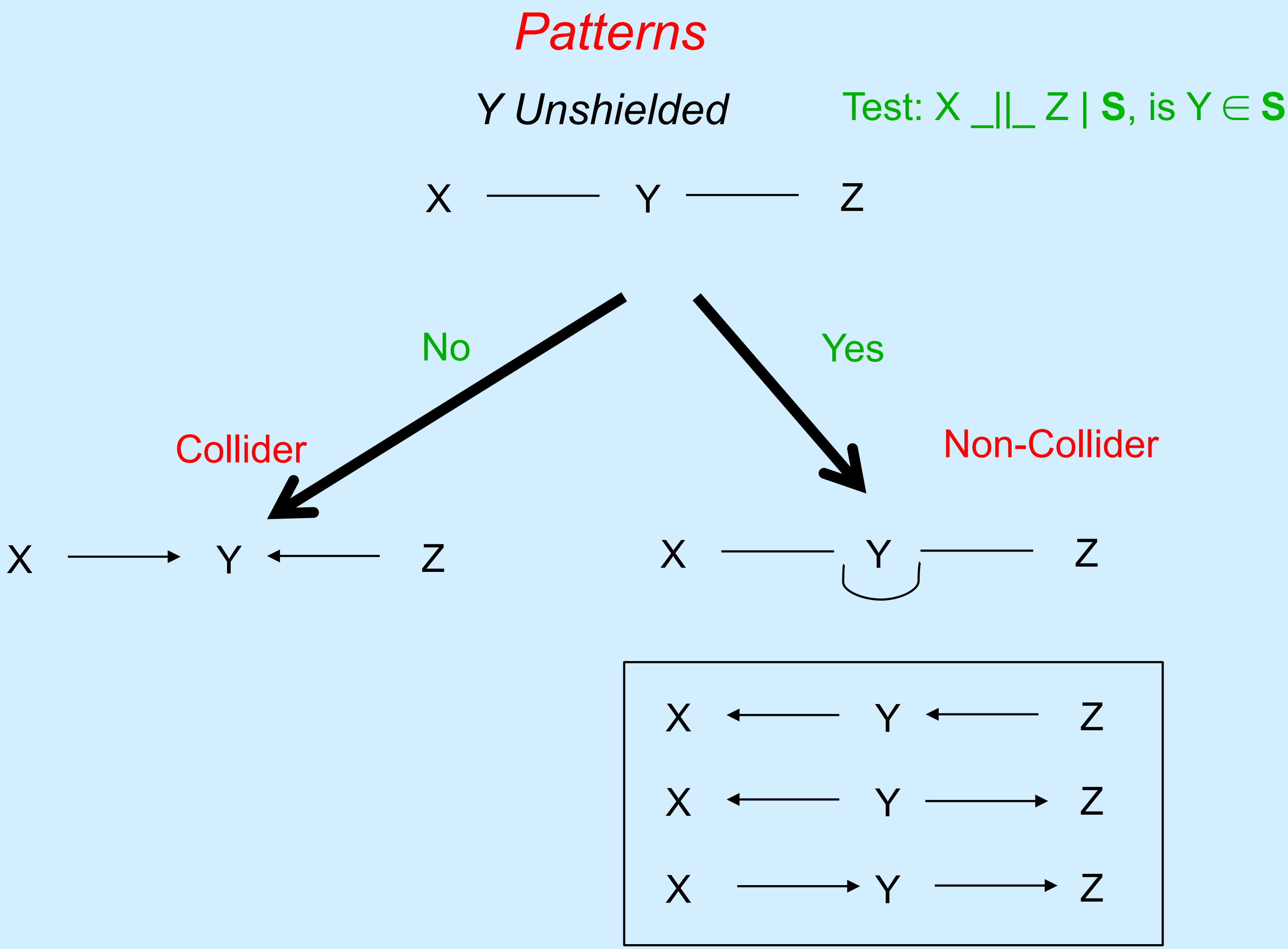
- Collider test:
  Find triples X – Y – Z, orient according to whether the set that separated X-Z contains Y

- Away from collider test:
  Find triples X → Y – Z, orient Y – Z connection via collider test

- Repeat until no further orientations

- Apply Meek Rules

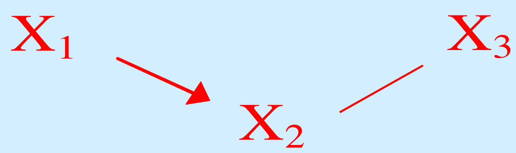
Search: Orientation



Search: Orientation

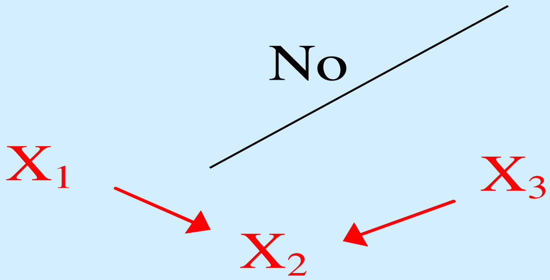
Away from Collider

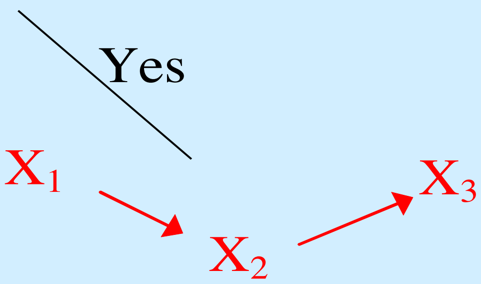
Test Conditions



1) X_1 - X_2 adjacent, and *into* X_2.
2) X_2 - X_3 adjacent
3) X_1 - X_3 not adjacent

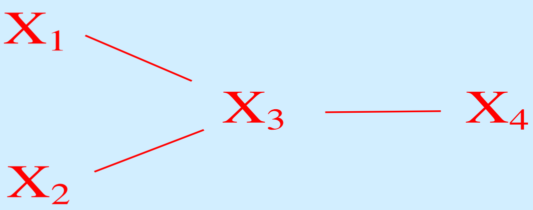
Test $X_1 \perp\!\!\!\perp X_3 \mid \mathbf{S}, X_2 \in \mathbf{S}$





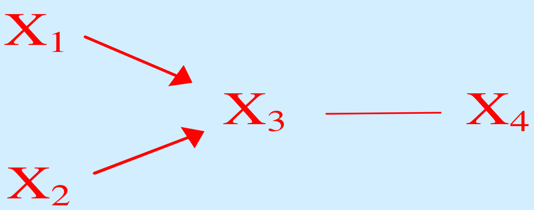
Search: Orientation

After Adjacency Phase



Collider Test: X1 – X3 – X2

X1 _||_ X2

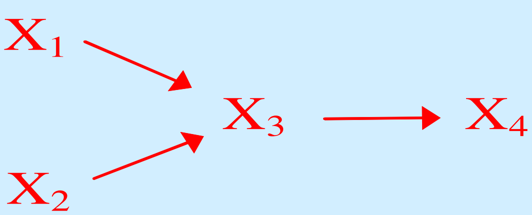


Away from Collider Test:

X1 → X3 – X4 *X2 → X3 – X4*

X1 _||_ X4 | X3

X2 _||_ X4 | X3



Away from Collider Power!

$X_1 \longrightarrow X_2 \;—\; X_3$ $\qquad$ $X_1 \,\_\|\_\, X_3 \mid \mathbf{S}, X_2 \in \mathbf{S}$

$X_1 \longrightarrow X_2 \longrightarrow X_3$

$X_2 – X_3$ oriented as $X_2 \rightarrow X_3$

Why does this test also show that X_2 and X_3 are *not confounded*?

$X_1 \longrightarrow X_2 \longrightarrow X_3$

$$\begin{tikzcd} & & C \arrow[dl] \arrow[dr] & \\ X_1 \arrow[r] & X_2 \arrow[rr] & & X_3 \end{tikzcd}$$

$X_1 \,\_\|\_\, X_3 \mid \mathbf{S}, X_2 \in \mathbf{S}$

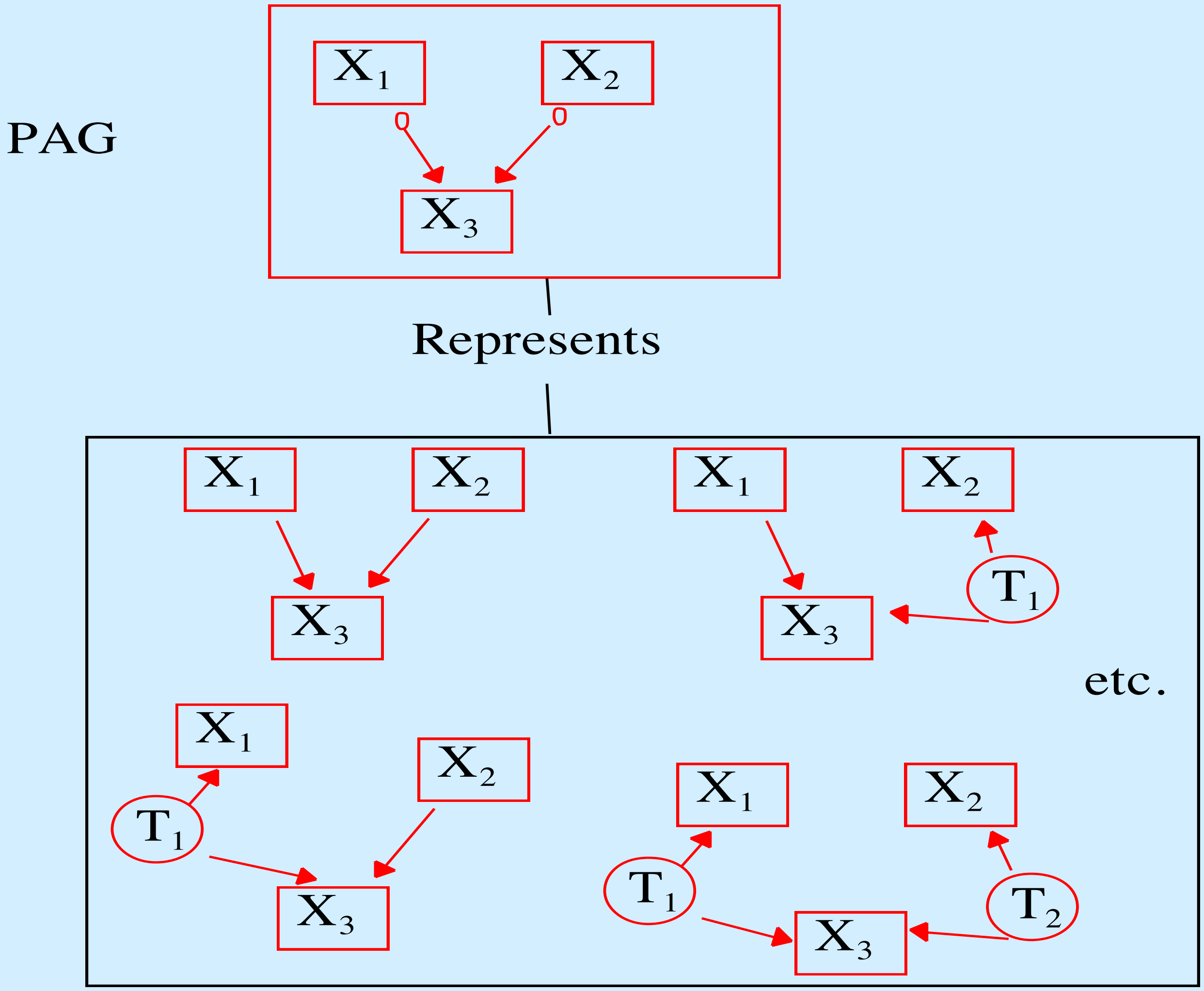
$X_1 \,\_\|\_\, X_3 \mid \mathbf{S}, X_2 \in \mathbf{S}, \mathbf{C} \notin \mathbf{S}$

Independence Equivalence Classes: Patterns & PAGs

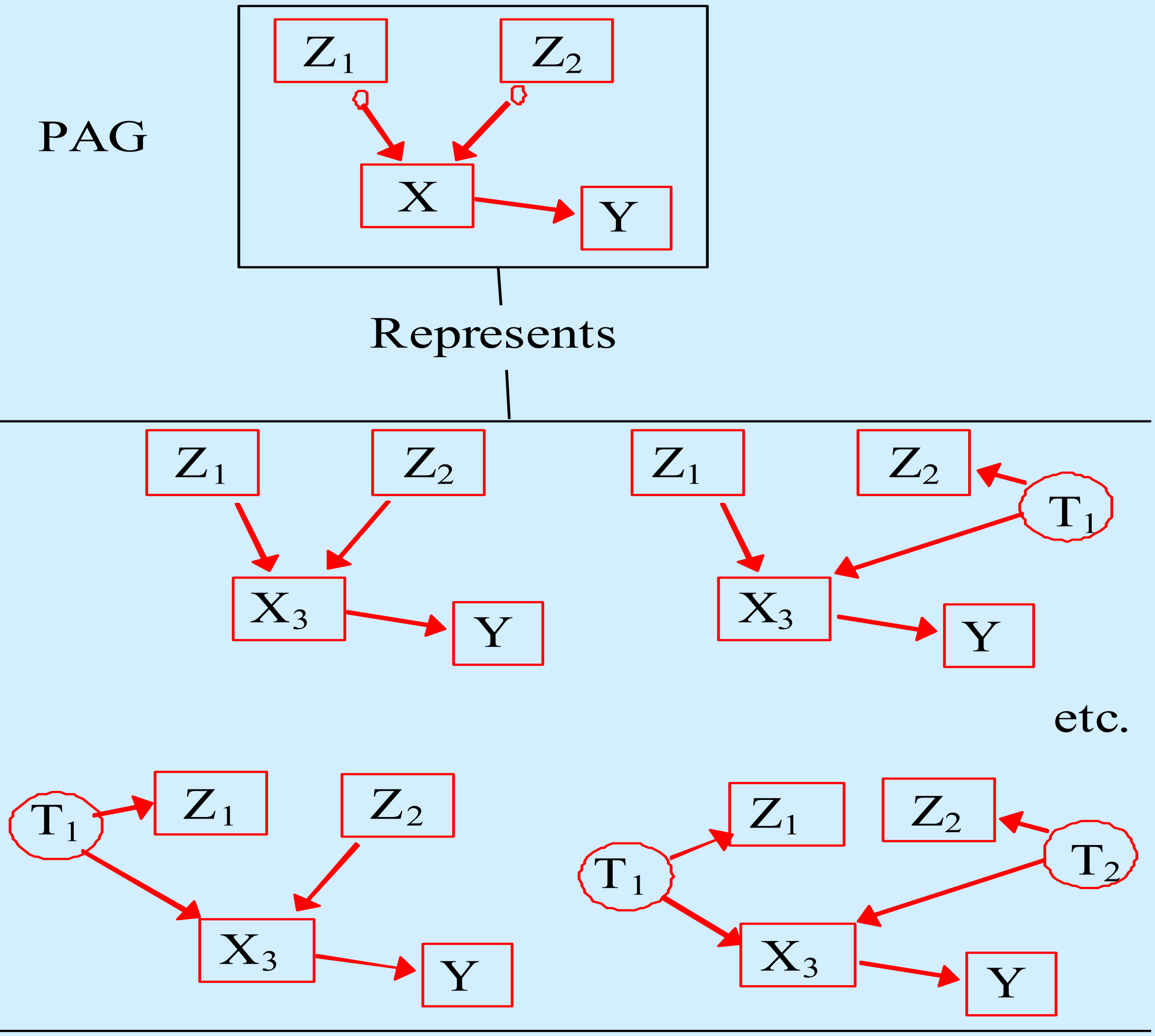
- Patterns (Verma and Pearl, 1990): graphical representation of d-separation equivalence among models with no latent common causes

- PAGs: (Richardson 1994) graphical representation of a d-separation equivalence class that includes models with *latent common causes* and *sample selection bias* that are d-separation equivalent over a set of measured variables **X**

PAGs: Partial Ancestral Graphs



PAGs: Partial Ancestral Graphs



PAGs: Partial Ancestral Graphs

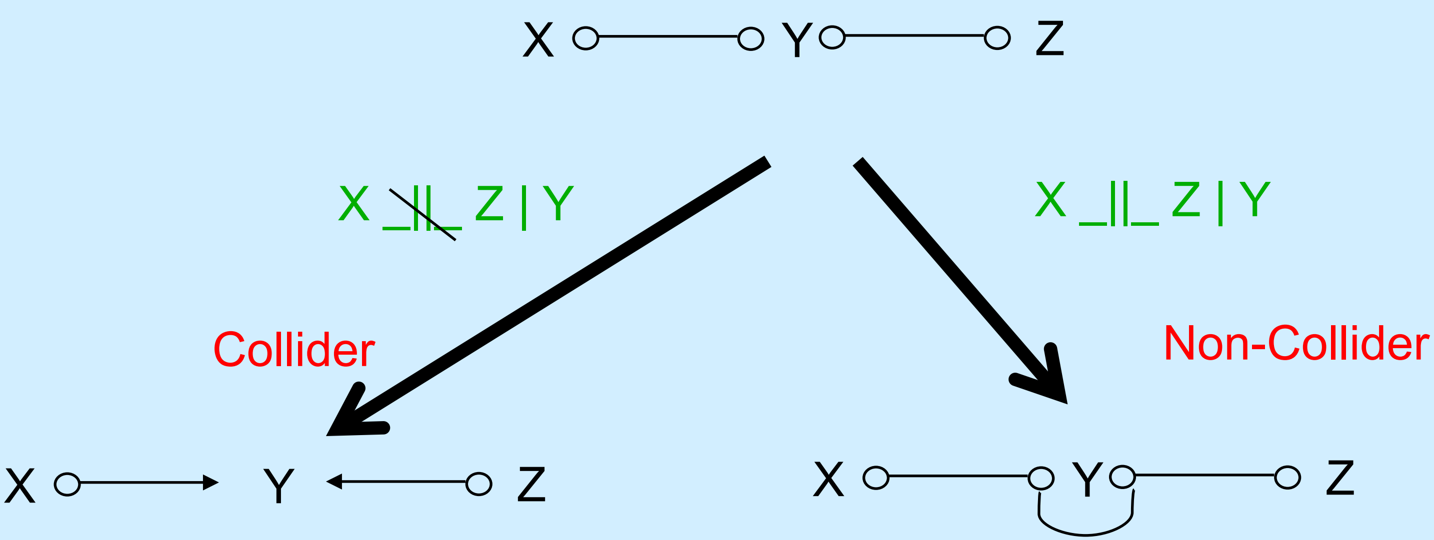
What PAG edges mean.

$X_1 \quad\quad X_2$	X_1 and X_2 are not adjacent
$X_1 \circ\!\!\rightarrow X_2$	X_2 is not an ancestor of X_1
$X_1 \circ\!\!-\!\!\circ X_2$	No set d-separates X_2 and X_1
$X_1 \rightarrow X_2$	X_1 is a cause of X_2
$X_1 \leftrightarrow X_2$	There is a latent common cause of X_1 and X_2

PAG Search: Orientation

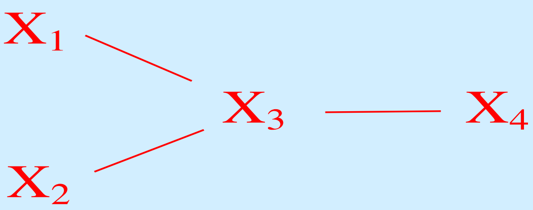
PAGs

Y Unshielded



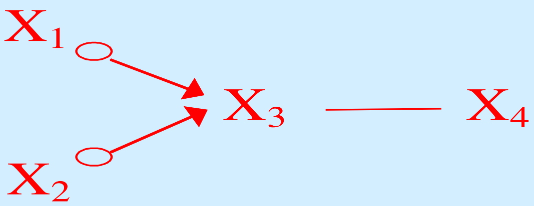
PAG Search: Orientation

After Adjacency Phase



Collider Test: X1 – X3 – X2

X1 _||_ X2

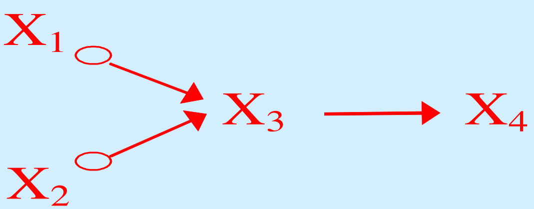


Away from Collider Test:

X1 → X3 – X4   X2 → X3 – X4

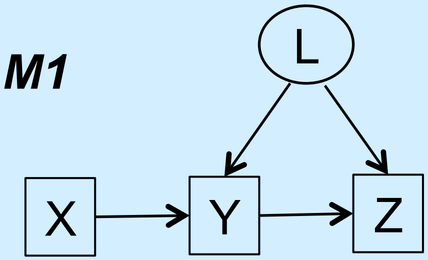
X1 _||_ X4 | X3

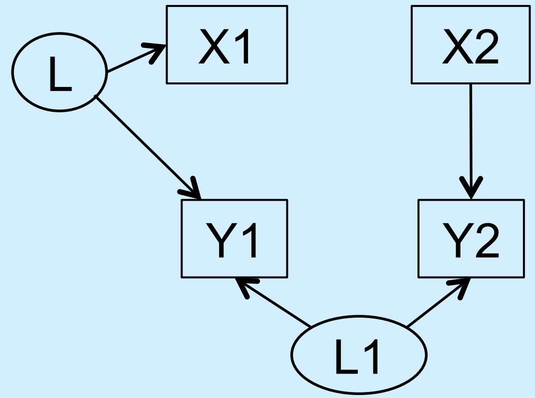
X2 _||_ X4 | X3



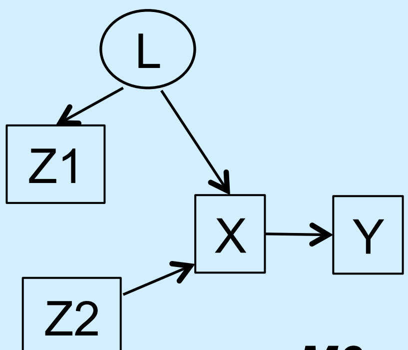
Interesting Cases

M1

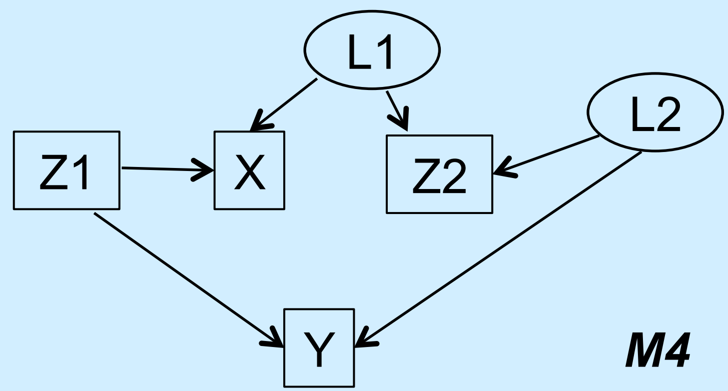




M2



M3

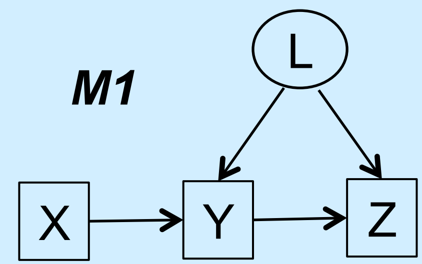


M4

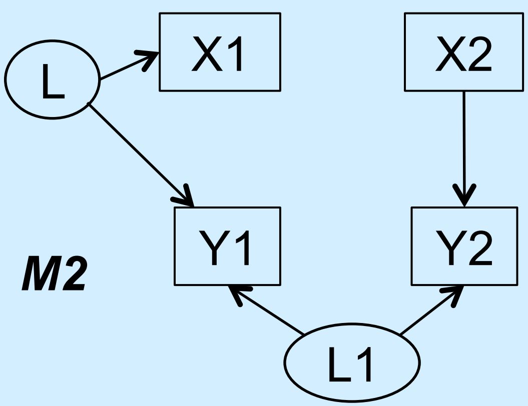
Tetrad Demo and Hands-on

1) Create new session
2) Select “Search from Simulated Data” from Template menu
3) Build graphs for M1, M2, M3 “interesting cases”, parameterize, instantiate, and generate sample data N=1,000.
4) Execute PC search, α = .05
5) Execute FCI search, α = .05

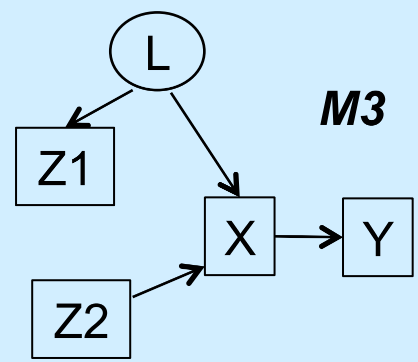
M1



M2



M3



Regression

&

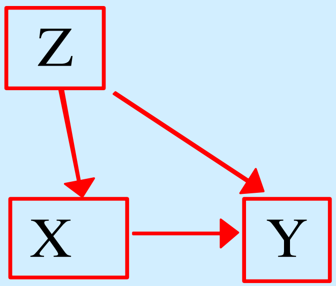
Causal Inference

Regression & Causal Inference

Typical (non-experimental) strategy:

1. Establish a prima facie case (X associated with Y)

But, omitted variable bias



2. So, identifiy and measure potential confounders **Z:**
   a) prior to X,
   b) associated with X,
   c) associated with Y

3. Statistically adjust for **Z** (multiple regression)

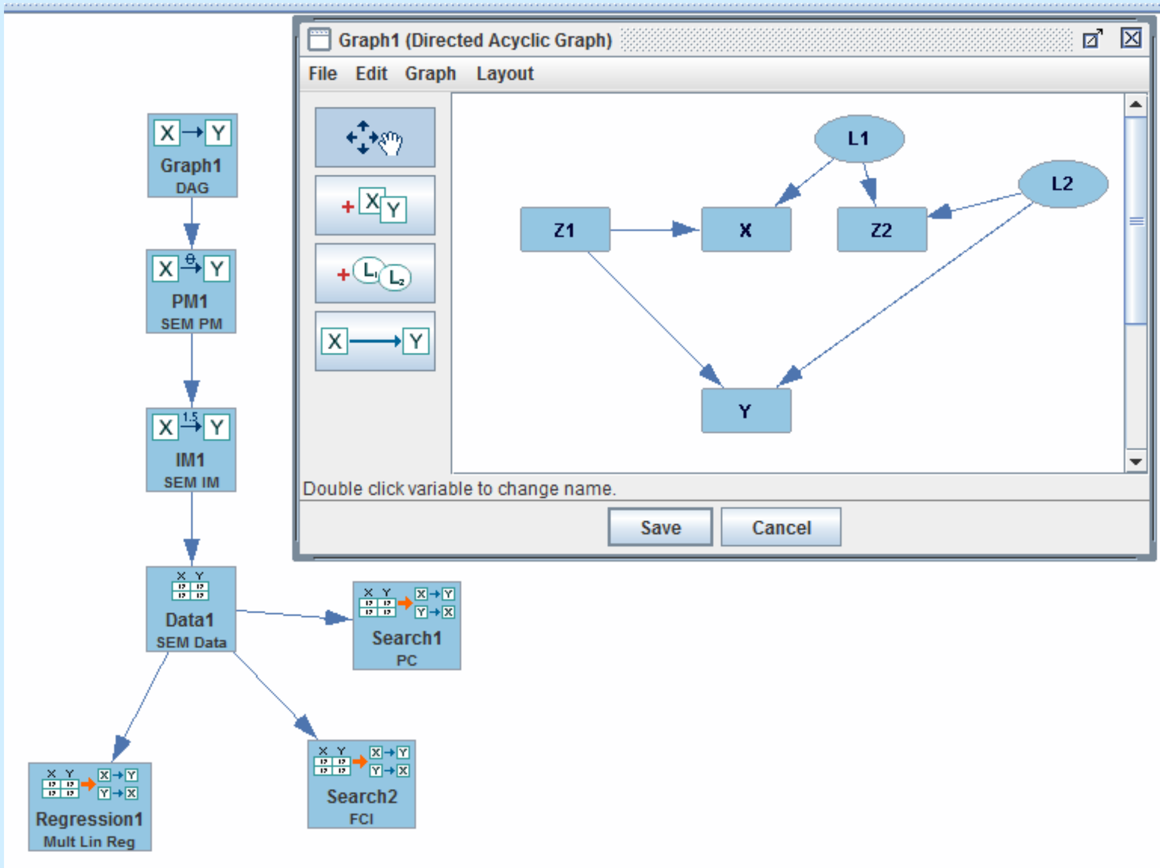
Regression & Causal Inference

Multiple regression or any similar strategy is provably unreliable for causal inference regarding X → Y, with covariates **Z**, unless:

- X prior to Y
- X, **Z**, and Y are causally sufficient (no confounding)

Tetrad Demo and Hands-on

1) Create new session
2) Select “Search from Simulated Data” from Template menu
3) Build a graph for M4 “interesting cases”, parameterize as SEM, instantiate, and generate sample data N=1,000.
4) Execute PC search, α = .05
5) Execute FCI search, α = .05



Summary of Search

Causal Search from Passive Observation

- PC, GES → *Patterns* (Markov equivalence class - no latent confounding)
- FCI → *PAG*s (Markov equivalence - including confounders and selection bias)
- CCD → Linear cyclic models (no confounding)
- BPC, FOFC, FTFC → (Equivalence class of linear latent variable models)
- Lingam → unique DAG (no confounding – linear non-Gaussian – faithfulness not needed)
- LVLingam → set of DAGs (confounders allowed)
- CyclicLingam → set of DGs (cyclic models, no confounding)
- Non-linear additive noise models → unique DAG
- Most of these algorithms are pointwise consistent – uniform consistent algorithms require stronger assumptions

Causal Search from Manipulations/Interventions

What sorts of manipulation/interventions have been studied?

- Do(X=x) : replace P(X | parents(X)) with P(X=x) = 1.0
- Randomize(X): (replace P(X | parents(X)) with $P_M(X)$, e.g., uniform)
- Soft interventions (replace P(X | parents(X)) with P_M(X | parents(X), I), P_M(I))
- Simultaneous interventions (reduces the number of experiments required to be guaranteed to find the truth with an independence oracle from N-1 to 2 log(N)
- Sequential interventions
- Sequential, conditional interventions
- Time sensitive interventions

Tetrad Demo and Hands-on

1) Search for models of Charitable Giving
2) Search for models of Foreign Investment
3) Search for models of Lead and IQ